PRESSURE AND VELOCITY DISTRIBUTIONS IN A VAPOR FLOW THROUGH NARROW ORIFICES DURING SUBLIMATION UNDER VACUUM

P. A. Novikov, G. L. Malenko, L. Ya. Lyubin, and V. I. Balakhonova

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Analytical relations are derived for the pressure distribution in an orifice between two sublimating disks. It is found that, in an isothermal flow of vapor, the excess pressure at the orifice center is determined by the orifice height and by the total ambient pressure.

In many types of engineering apparatus and evaporator–sublimator heat exchangers one of the major design goals is their compactness. The vapor is removed here often through narrow orifice channels. It then becomes necessary to determine the limiting distance between heat-and-mass transfer surfaces at which the thermodynamic characteristics of the medium inside the orifice would still not change drastically and the technological process would still continue under favorable conditions. With this in view, the authors have made a theoretical and experimental study of the sublimation process in narrow orifices between two flat disks in a rarefied gas atmosphere.

1. We consider a narrow orifice of height 2h formed by two flat disks of diameter $2r_0$, with vapor generated by sublimation at their inside surfaces at a constant rate $j_m = v_z\rho = \text{const}$ (Fig. 1). The temperature of these inside surfaces is considered constant $T = T_s = \text{const}$. Thus, we are dealing with a laminar isothermal flow of gas in the gap between disks (viscous flow of a rarefied gas) characterized by a small Reynolds number $Re = \rho V_0 h^2/(\mu r_0)$, as in the case of a Hiu-Shou flow [1]. Since $h/r_0 \ll 1$, hence $v_z \ll v_r$ within a small region around the z-axis and the equation of motion for the gas can be replaced by the approximate equation:

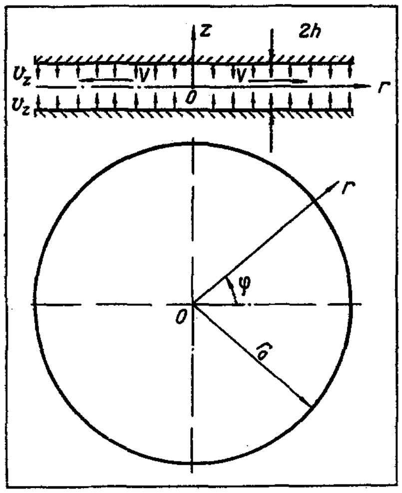

Fig. 1. Schematic diagram of a horizontal gap with proper notation.

$$\mu\frac{\partial^2 v_r}{\partial z^2} - \frac{\partial P}{\partial r} = \rho v_r\frac{\partial v_r}{\partial r} + \rho v_z\frac{\partial v_r}{\partial z}. \quad (1)$$

In this case it is permissible to neglect the change in pressure and, therefore, also the change in density within distances comparable with the orifice height, which allows us to write the continuity equation as

$$\frac{\partial v_r}{\partial r} + \frac{v_r}{r} + \frac{\partial v_z}{\partial z} = 0. \quad (2)$$

The Knudsen number corresponding to this flow is assumed to have sufficiently small values. Therefore, $v_r = 0$ at $z = \pm h$.

The right-hand side of Eq. (1) is a quantity of the order of Re. If the solution is sought in the form of a series in the small parameter Re (for brevity, we do not introduce dimensionless variables in this analysis), therefore, then for the linear approximation to Eq. (1) we have

$$v_r = v_{r1} = \frac{3}{2}V_1\left(1 - \frac{z^2}{h^2}\right), \quad V_1 = -\frac{h^2}{3\mu}\cdot\frac{dP_1}{dr}. \quad (3)$$

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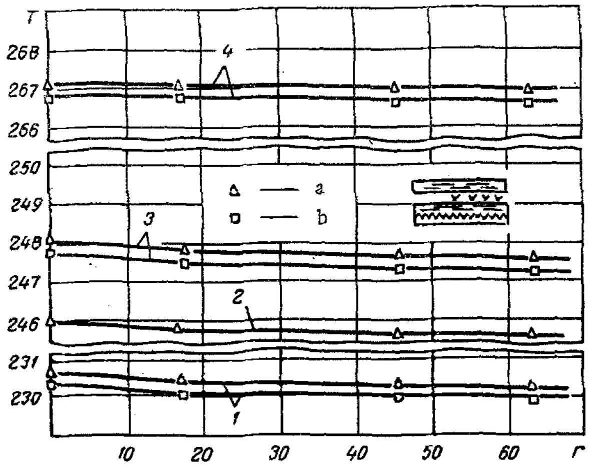

Fig. 2. Variation of the vapor temperature T (°K) over the orifice length r (mm): 1) P = 5.33 N/m^2 and $J_m = 0.49 \cdot 10^{-4}$ kg/m^2 · sec; 2) P = 133.3 N/m^2 and $J_m = 1.16 \cdot 10^{-4}$ kg/m^2 · sec; 3) P = 133.3 N/m^2 and $J_m = 0.95 \cdot 10^{-4}$ kg/m^2 · sec; 4) P = 266.6 N/m^2 and $J_m = 0.968 \cdot 10^{-4}$ kg/m^2 · sec; a) 2h = 2 mm; b) 5 mm.

Component v_z will be found from Eq. (2):

$$v_z = \frac{3}{2}\left(\frac{dV_1}{dr} + \frac{V_1}{r}\right)\left(z - \frac{1}{3}\frac{z^3}{h^2}\right). \tag{4}$$

In order to determine the pressure distribution between the sublimating disks, we use the equation of mass balance and the equation of state:

$$\frac{d(r\rho V)}{dr} = \frac{rj_m}{h},\ \rho = \frac{P}{RT}, \tag{5}$$

from where

$$V_1 = rj_mRT/(2hP_1). \tag{6}$$

Thus, taking into account (3), we obtain in the linear approximation

$$P_1\frac{dP_1}{dr} = -\frac{3}{2}\cdot\frac{\mu j_m RT}{h^3}r,$$

i.e.,

$$P_1^2 = P_0^2 + \frac{3}{2}\cdot\frac{\mu j_m RT}{h^3}(r_0^2 - r^2). \tag{7}$$

As the Reynolds number increases, the velocity distribution (3) becomes somewhat distorted and this, in turn, has an effect on the pressure distribution. For an estimate of this effect we make the following iteration:

$$v_r = v_{r1} + v_{r2} + \cdots,\ P = P_1 + P_2 + \cdots,\ V = V_1 + V_2 + \cdots$$

Inserting (3) and (4) into (1) yields

$$\frac{\partial^2 v_{r2}}{\partial z^2} = \frac{1}{\mu}\cdot\frac{dP_2}{dr} + \frac{9}{4}\cdot\frac{\rho_1}{\mu}V_1\frac{dV_1}{dr}\left(1 - \frac{z^2}{h^2}\right)^2 - \frac{9}{2}\cdot\frac{\rho_1}{\mu}V_1\left(\frac{dV_1}{dr} + \frac{V_1}{r}\right)\left(\frac{z^2}{h^2} - \frac{1}{3}\cdot\frac{z^4}{h^4}\right).$$

Consequently,

$$v_{r2} = \frac{1}{\mu}\cdot\frac{dP_2}{dr}\cdot\frac{z^2 - h^2}{2} + \frac{9}{4}\cdot\frac{\rho_1}{\mu}V_1\frac{dV_1}{dr}\left(\frac{z^2 - h^2}{2} - \frac{z^4 - h^4}{6h^2} + \frac{z^6 - h^6}{30h^4}\right) - \frac{9}{2}\cdot\frac{\rho_1}{\mu}V_1\left(\frac{dV_1}{dr} + \frac{V_1}{r}\right)\left(\frac{z^4 - h^4}{12h^2} - \frac{z^6 - h^6}{90h^4}\right).$$

The mean-over-the-height velocity in the orifice is

$$V_{r_2} = -\frac{h^2}{3\mu}\cdot\frac{dP_2}{dr} - \frac{9}{35}\cdot\frac{h^2}{\mu}\rho_1\left(V_1\frac{dV_1}{dr} + \frac{V_1^2}{r}\right). \tag{8}$$

Following Eq. (1) of mass balance,

$$d[r(\rho_1 V_2 + \rho_2 V_1)]/dr = 0,$$

i.e., the regular component V_2 at r = 0 is determined from the relation

$$\rho_1 V_2 + \rho_2 V_1 = 0.$$

Inserting here expressions (6), (7), and (8) leads to the equation:

$$\frac{dP_2}{dr} - A_1(r)P_2 + A_2(r) = 0, \tag{9}$$

where

$$A_1 = \frac{3\mu j_m RT}{2h^3}r\left[P_0^2 + \frac{3}{2}\cdot\frac{\mu j_m RT}{h^3}(r_0^2 - r^2)\right]^{-1},$$

$$A_2 = \frac{81}{280}\cdot\frac{\mu j_m^3 R^2 T^2}{h^5}r^3\left[P_0^2 + \frac{3}{2}\cdot\frac{\mu j_m RT}{h^3}(r_0^2 - r^2)\right]^{-3/2}.$$

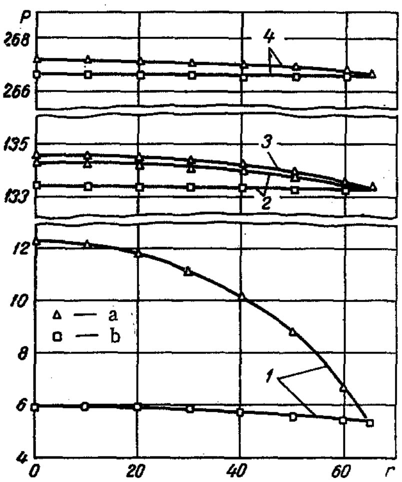

Fig. 3

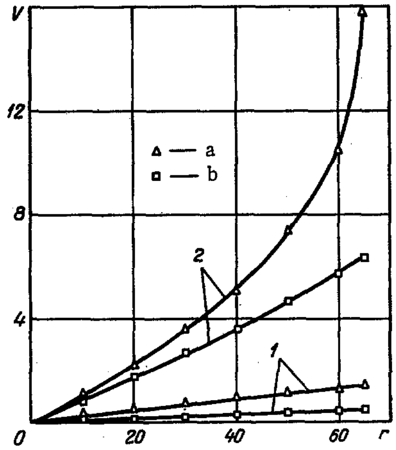

Fig. 4

Fig. 3. Variation of the vapor pressure P (N/m^2) over the orifice length r (mm): 1) P = 5.33 N/m^2 and $J_m = 0.49 \cdot 10^{-4}$ $kg/m^2 \cdot sec$; 2) P = 133.3 N/m^2 and $J_m = 0.95 \cdot 10^{-4}$ $kg/m^2 \cdot sec$; 3) P = 133.3 N/m^2 and $J_m = 1.16 \cdot 10^{-4}$ $kg/m^2 \cdot sec$; 4) P = 266.6 N/m^2 and $0.968 \cdot 10^{-4}$ $kg/m^2 \cdot sec$; a) 2h = 2 mm; b) 5 mm.

Fig. 4. Variation of the vapor velocity V (m/sec) over the orifice length r (mm): 1) P = 133.3 N/m^2 and $J_m = 0.95 \cdot 10^{-4}$ $kg/m^2 \cdot sec$; 2) P = 5.33 N/m^2 and $J_m = 0.99 \cdot 10^{-4}$ $kg/m^2 \cdot sec$; a) 2h = 2 mm; b) 5 mm.

Since $P_2 = 0$ at $r = r_0$, a solution of (9) yields

$$P_2(r) = -\frac{9}{140} \cdot \frac{j_m h}{\mu P_1(r)} \left[2P_1^2(0) \ln \frac{P_1(r_0)}{P_1(r)} + \frac{3}{2} \cdot \frac{\mu j_m RT}{h^3} (r_0^2 - r^2) \right]. \quad (10)$$

2. In order to evaluate the effect of an anisothermal flow on the sublimation process, we consider the following case. A sublimating surface receives heat from sources located inside ice disks at a distance δ away from it. The temperature of these heaters is assumed constant. We will also assume that the radial temperature distribution is the same along the inside disk surfaces and within the vapor volume and corresponds to the conditions of phase equilibrium, according to the Clapeyron–Clausius equation, i.e., is approximately

$$\frac{T(r)}{T(0)} = \frac{1}{1 - [RT(0)/\Lambda] \ln [P(r)/P(0)]}. \quad (11)$$

In the case of substances with an appreciable heat of sublimation, which are actually used in sublimators, one may disregard the effect which the heat supplied to the disks from an expanding vapor has on the sublimation process. Therefore, the sublimation rate will be determined from the relation:

$$j_m(r) = j_m(0) - \frac{\lambda}{\Lambda\delta} [T(r) - T(0)]. \quad (12)$$

We consider now only the limiting case where $Re \to 0$ ($P \to P_1$, $V \to V_1$). Then, by virtue of (3) and (5),

$$\frac{d^2P}{dr^2} + \left(\frac{1}{\rho} \cdot \frac{d\rho}{dr} + \frac{1}{r} \right) \frac{dP}{dr} + \frac{3\mu}{h^3} \cdot \frac{j_m}{\rho} = 0. \quad (13)$$

Inserting (11) and (12) into (13), we obtain

$$\frac{d^2P}{dr^2} + \left\{ [1 - F(P)] \frac{1}{P} \cdot \frac{dP}{dr} + \frac{1}{r} \right\} \frac{dP}{dr} + \frac{3\mu\Lambda}{h^3} \cdot \frac{F(P)}{P} \left[j_m(0) - \frac{\lambda T(0)}{\Lambda\delta} F(P) \right] = 0, \quad (14)$$

where

$$F(P) = \frac{RT(0)/\Lambda}{1 - [RT(0)/\Lambda] \ln [P(r)/P(0)]}.$$

The pressure distribution in the orifice is found by solving the last equation with the following boundary conditions:

$$dP/dr = 0 \text{ for } r = 0,\ P = P_0 \text{ for } r = r_0.$$

In analyzing the sublimation process over a rather wide range of pressures, however, it becomes worthwhile to replace the solution to this nonlinear boundary-value problem by a solution to the Cauchy problem, which we will obtain if the vapor pressure at the center of the disks P(0) is given. The radial pressure distribution is then easily calculated with the aid of a computer.

In order to verify the analytical results, we have made an experimental study of water vapor flowing through a narrow orifice between two disks 130 mm in diameter at various ambient pressures (5.33-266.6 N/m^2). The lower disk was covered on the orifice side with a layer of ice (H_2O). Heat to the sublimating surface was supplied by means of an electric heater 130 mm in diameter built into the ice disk. The end surfaces of the ice disk were carefully insulated. After assembly, the disk was placed on a laboratory scale for measuring the loss of weight during the sublimation process. The upper disk was adjusted vertically by means of an electric drive.

The necessary orifice height 2h was established automatically from the control panel inside the vacuum chamber. This eliminated the possibility of losing vacuum during an adjustment of the orifice height and, therefore, reduced the dispersion of measured values. The orifice height was varied from 2 to 20 mm. The vapor temperature along the orifice was measured with three copper – constantan differential thermocouples installed along the disk radius. With the aid of a refrigerator, the upper disk and the ambient medium as well as the walls of the vacuum chamber and other equipment were maintained at the temperature of the sublimating ice surface on the lower disk.

The distribution of the vapor temperature over the orifice length (Fig. 2) indicates only a small temperature variation near the axis (within a 10 mm radius). Within this region the temperature remains almost constant, i.e., the assumption made in paragraph 1 concerning a uniform temperature distribution in the orifice has been shown by this experiment to be valid. Therefore, the vapor flow through this orifice may be treated as an isothermal process and the analytical relations derived earlier are applicable.

In our experiment the Reynolds number was varied within the Re = 0.003-0.030 range and, consequently, the pressure distribution in the orifice could be determined according to Eq. (7). Generation of sublimate vapor at the surface of one disk only was considered in the calculations.

According to Fig. 3, the largest pressure drop along the orifice occurred when the pressure in the chamber was 5.33 N/m^2 and the orifice height was 2h = 2 mm (curve 1a). Characteristically, the variation in the vapor pressure was affected primarily by the orifice height and not by the evaporation rate J_m. A 22% increase in J_m at a pressure of 133.3 N/m^2 in the chamber, for example, raised the vapor pressure at the center of the 2h = 2 mm orifice by 0.16% (curves 2 and 3) and only by 0.01% at the center of the 2h = 5 mm orifice. Furthermore, the absolute drop of vapor pressure at the center of as well as along the orifice decreased as the total pressure in the chamber and and as the orifice height increased (curves 1 and 4). As the orifice height was increased further (2h > 5 mm), the pressure along the orifice remained almost equal to the pressure inside the chamber and, therefore, the distribution here is not shown. The density of vapor flowing along the orifice followed an analogous trend.

The distribution of the mean-over-the-section vapor velocity was calculated according to formula (6), as shown in Fig. 4. At chamber pressures within the 133.3-266.6 N/m^2 range the vapor velocity varied almost linearly, while at a pressure of 5.33 N/m^2 the velocity increased from the center toward the periphery more rapidly (curves 2) as a result of a more rapid variation of the pressure drop along the orifice following a reduction of the total ambient pressure.

Thus, the analytical and the experimental results indicate that, in an isothermal flow of vapor through a narrow orifice, the most important factors which affect the distribution of pressures and velocities along the orifice are the orifice height and the total ambient pressure. For example, a decrease in the orifice height down to 1 mm (h/r_0 = 0.001, pressure P = 5.33 N/m^2) makes the pressure at the orifice center over 6-7 times higher than at the periphery. Such a significant pressure rise may result in a corresponding temperature rise at the center of a specimen and may disturb the normal process mode. This becomes particularly important in sublimator-type heat exchangers operating on the principle of cooling by evaporation from pores under vacuum, where the surface temperature of porous elements varies narrowly between 271 and 273°K even under wide fluctuations of the heat load [2]. For this reason, increasing the pressure at the center between two such elements may result in the melting of the ice and an ejection of liquid from the porous elements.

Thus, in designing a heat exchanger aggregate consisting of several flat elements (e.g., in [3]) it is important in each specific case to consider not only the total ambient pressure but, above all, the orifice height 2h and radius r as well as the vapor flow rate.

NOTATION

J_m	is the sublimation rate;
2h	is the orifice height;
$2r_0$	is the orifice diameter;
T, P, ρ	are the temperature, pressure, and density of vapor in the orifice;
μ	is the dynamic viscosity of vapor;
v_r, v_z	are the radial and transverse component of velocity;
V	is the mean-over-the-length vapor velocity in the orifice;
λ	is the thermal conductivity of ice;
δ	is the distance from the heater to the sublimating surface;
A	is the latent heat of sublimation;
R	is the gas constant;
$Re = V(r_0)h^2/(\nu r_0)$	is the Reynolds number characterizing the vapor flow through the orifice.

Subscript

0 refers to the orifice center.

LITERATURE CITED

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